LAGEOS III AND THE GRAVITOMAGNETIC FIELD

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I. IMPORTANCE OF THE GRAVITOMAGNETIC FIELD

a)   A Never-measured Field of Nature

In electrodynamics, in the frame at rest with an electrically charged sphere we have an electric field. If we then rotate the sphere we observe a magnetic field, which strength is proportional to the angular velocity.

Similarly, in Einstein geometrodynamics (Misner *et al.* 1973, Wheeler 1964) (general relativity), a non-rotating, massive sphere produces the standard Schwarzschild field. If we then rotate the sphere we have the occurrence of the gravitomagnetic field, whose strength, in the weak field limit, is proportional to the sphere angular velocity. In the weak field approximation of the Kerr metric, the gravitomagnetic field is (Thorne *et al.* 1986):

$$\vec{H} = \vec{\nabla} \times \vec{\beta} \cong 2 \left[\frac{\vec{J} - 3 (\vec{J} \cdot \hat{r}) \hat{r}}{r^3} \right] \tag{1}$$

where $\vec{\beta} \cong (0,0, -\frac{2J}{r^3})$ is the gravitomagnetic or Lense-Thirring potential and J is the angular momentum of the central body.

We observe that gravitomagnetism and gravitational waves are the two main aspects, still to be measured, of Einstein geometrodynamics analogous to magnetic field and electromagnetic waves of electrodynamics.

b)   At the Foundations of Inertia in Einstein General Relativity

In Einstein geometrodynamics (Misner *et al.* 1973, Wheeler 1964), to solve the initial value problem (York 1979, Choquet-Bruhat and York 1980) as a part of the initial conditions, we need to specify on a Cauchy hypersurface Σ the conformal mass-energy current density: $\vec{j}$ base. Through the initial conditions, with the field equations, we then solve for the geometry $g_{\alpha\beta}$ of the universe and eventually, we determine the local inertial frames (where $g_{\alpha\beta} \rightarrow \eta_{\alpha\beta}$) all over the spacetime. The axes of the local inertial frames (gyroscopes) are therefore influenced, and partially determined, by the mass-energy currents in the universe (dragging of inertial frames).

This agrees with some kind of general relativistic formulation of the Mach principle, according to the ideas of Einstein and Wheeler (Wheeler 1988, Carifolini and Wheeler 1987). "Nothing would do more to demonstrate the inertia-influencing effect of mass in motion than to detect and measure the Einstein-Thirring-Lense-predicated gravitomagnetism of the earth" (Wheeler 1988).

c) *A Key Role in Theories of Quasars and Active Galactic Nuclei*

In high-energy astrophysics, some theories of energy storage, power generation, jet formation and jet alignment of quasars and active galactic nuclei are based on the existence of the gravitomagnetic field of a supermassive black hole (Thorne *et al.* 1986).

In particular, this field may explain the constant direction of the jets over millions of light years and therefore over a time of several millions of years. Through the standard Navier-Stokes equations, in which one includes the gravitomagnetic field of the central object, one can show (Bardeen and Petterson 1975) that the accretion disk tends to be oriented into the equatorial plane of the central body — Bardeen-Petterson effect. The jets are then ejected normally to the accretion disk, that is, normally to the equatorial plane of the central body. The angular momentum vector of the central black hole acts, therefore, as a gyroscope and this may explain the constant direction of the jets.

II. LAGEOS III
LASER RANGED SATELLITES TO DETECT THE GRAVITOMAGNETIC FIELD
AND SUPPLEMENTARY INCLINATION SATELLITES
TO AVOID GRAVITY FIELD UNCERTAINTIES

Many experiments have been proposed to measure the gravitomagnetic field.

The GPB experiment (Everitt 1974, Lipa *et al.* 1974) intends to measure the Lense-Thirring-Schiff (Lense and Thirring 1918) precession of gyroscopes orbiting the earth.

Polar satellites have been proposed to measure the Lense-Thirring precession of the orbital plane — an enormous gyroscope (Schiff 1960, Yilmaz 1959) and two guided, drag-free, counter-rotating, polar satellites have been suggested to avoid orbital inclination errors (Van Patten and Everitt 1976). We recall that a polar satellite: $I \cong 90°$, has a null classical nodal precession.

Here, we briefly summarize the new idea (Ciufolini 1985, 1984, 1987, 1988) to measure the gravitomagnetic drag of the nodes of two nonpolar, supplementary inclination, laser ranged satellites. This idea can be decomposed into two parts:

(1)   Position measurements of laser ranged satellites, of LAGEOS (Smith and Dunn 1980, Yoder *et al.* 1983, Cohen *et al.* 1985) type, are accurate enough to detect the tiny effect due to the gravitomagnetic field: the Lense-Thirring precession.

(2)   To cancel out the uncertainties in the enormous perturbations due to the nonsphericity of the earth gravity field, we need (Ciufolini 1985, 1984, 1987) a new satellite, LAGEOS III, with semimajor axis and eccentricity, a and e, equal to those of LAGEOS, but with supplementary inclination: $I_{III} \cong 70°$.

The LAGEOS (Smith and Dunn 1980, Yoder *et al.* 1985) semimajor axis is a = 12270 km, the period P = 3.758 h, the eccentricity e = 0.004 and the inclination I = 109.94°. The period of the node $P(\Omega) = 1046$ days.

The Lense-Thirring nodal precession is for LAGEOS (Ciufolini 1984).

$$\dot{\Omega}^{Lense}_{Thirring \; LAGEOS} = \frac{2GJ_{\oplus}}{c^2 a^3 (1-e^2)^{3/2}} \cong 31 \text{ milliarcsec/year} \qquad (2)$$

where $J_{\oplus} \cong 5.9 \times 10^{40}$ g $\cdot$ cm2/sec $\cong 1.5 \times 10^2$ cm^2 is the earth angular momentum.

Unfortunately, the Lense-Thirring precession cannot be extracted from the measured value of the LAGEOS nodal precession, because of the uncertainty in the theoretical classical precession due to the quadrupole and higher mass moments of the earth:

$$\dot{\Omega}^{Classical}_{LAGEOS} \cong 126°/\text{year}$$

However, a new satellite, of LAGEOS type: LAGEOS-III, with supplementary inclination: $I_{LAGEOS \; III} \cong 70°$, would have a classical precession equal in magnitude and opposite in sign to that of LAGEOS. By contrast, since independent of the inclination, the Lense-Thirring precession (2) would be the same, both in magnitude and sign for the two satellites. Therefore, from the sum of the measured nodal precessions, we should be able (Ciufolini 1985, 1984, 1987, 1988) to measure the Lense-Thirring effect.

III. ERROR SOURCES
ORBITAL INJECTION ERRORS, GRAVITATIONAL AND NONGRAVITATIONAL PERTURBATIONS AND MEASUREMENT ERRORS

(1)   Errors from gravitational perturbations arise from uncertainties in modeling the LAGEOS gravitational nodal perturbations. These errors may be subdivided into:

(a)   Errors from orbital injection errors (Ciufolini 1988) due to the uncertainties in the knowledge of the static part of the even zonal harmonic coefficients, J_{2n}, of the earth gravity field. These errors are, *a priori*, zero for the two supplementary inclination satellites. However, any deviation of the orbital parameters of LAGEOS III from the optimal values will introduce uncertainties which must be evaluated.

(b)   Errors from other gravitational perturbations (Ciufolini 1988). Static odd zonal harmonic perturbations; static nonzonal harmonic perturbations; nonlinear harmonic perturbations; solid and ocean earth tides. De Sitter (or geodetic) precession (Bertotti *et al.* 1987); sun, moon and planetary tidal accelerations; nonlinear, n-body, general relativistic effects; other very tiny relativistic deviations from geodesic motion of LAGEOS. The main error source is, however, due (Ciufolini 1987, 1988) to the uncertainties in modeling the dynamical part of the earth gravity field, that is, to the uncertainties in modeling solid and ocean earth tides.

(2)   Errors from nongravitational perturbations (Ciufolini 1987, 1988). Direct solar radiation pressure; earth albedo; satellite eclipses; anisotropic thermal

radiation; Poynting-Robertson effect; infrared radiation; atmospheric drag; solar wind; interplanetary dust; earth magnetic field. The main error sources are, however, due to uncertainties in modeling the earth albedo, anisotropic thermal radiation, and atmospheric drag (Ciufolini 1987, 1988).

(3)   Errors from measurement uncertainties in the orbital parameters (Ciufolini 1988). These errors are due to the uncertainties in the measurement of the LAGEOS orbital parameters and, in particular, to the errors in the measurement of the inclination I and the nodal longitude Ω, relative to an asympotic inertial frame (Ciufolini 1988).

IV. PRELIMINARY ERROR ANALYSIS
A PRELIMINARY 10% MAXIMUM ERROR,
OVER THE PERIOD OF THE NODE

A major problem in modeling the LAGEOS orbit is the average secular decrease (Cohen et al. 1985, Rubincam 1988) of the LAGEOS semimajor axis of about 1 mm per day. This corresponds to an along-track acceleration of about -3×10^{-10} cm/sec^2. This acceleration may be explained by three mechanisms (Rubincam 1988, Afonso et al. 1988):

(1)   Neutral and charged particle drag (Rubincam 1982, Afonso et al. 1985).

(2)   Thermal drag from the earth infrared radiation (Rubincam, 1987 and 1988) and the thermal lag of the LAGEOS retroreflectors, due to their thermal inertia. The re-emission causes an along-track acceleration opposite to the satellite motion.

(3)   Thermal drag from the sun radiation plus satellite eclipses by the earth (Afonso et al. 1988). The sun radiation is absorbed and then re-emitted by the satellite. This phenomenon may be important when the satellite orbits are partially in the shadow of the earth.

The effect of these three perturbations on the node has been investigated in relation to the supplementary inclination configuration (Ciufolini 1988, Farinella 1988). The preliminary result is that their effect should not be larger than 1% $\dot{\Omega}$ Lense-Thirring due to particle drag (Ciufolini 1987, 1988). This is negligible over the period of the node for two supplementary inclination satellites due to infrared radiation (Ciufolini 1988, Farinella 1988) and not larger than 2% $\dot{\Omega}$ Lense-Thirring due to thermal drag plus satellite eclipses (Ciufolini 1988).

To support these preliminary figures, we observe that to give a secular nodal precession, a force must: 1) be perpendicular to the orbital plane, and 2) change sign in half a period. Even in this worst case, an acceleration with amplitude $\sim 3 \times 10^{-10}$ cm/sec^2 can, at most, cause a LAGEOS nodal precession of 5% $\dot{\Omega}$ Lense-Thirring, as it is easily calculated (Ciufolini 1988) from the Lagrange equation for the node. Ciufolini (1988) has carried out a comprehensive preliminary analysis. The result is that, using 2 nonpolar, laser ranged satellites with supplementary inclinations, the maximum error, over the period of the node of

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~3 years, should not be larger than ~10% of the gravitomagnetic effect to be measured.

A study group, composed of University of Texas at Austin and Italian scientists, is, at present, performing a comprehensive computer simulation and covariance analysis of the experiment.

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DISCUSSION

SHAPIRO: LAGEOS-II will of course help to understand the partially unmodeled secular variation of the semimajor axis.

CIUFOLINI: Certainly; however this unmodeled variation should not be a problem for the gravitomagnetic experiment. In fact, even in the worst case that a comparable acceleration would be acting perpendicularly to the orbital plane and changing sign in half a period, that is, even in the case of a maximum contribution to the nodal rate, the total nodal drag due to this acceleration would not be more than 5% of the Lense-Thirring effect.

NORDTVEDT: Why do you quote a dJ_2/J_2 uncertainty limitation, when you might use the LAGEOS orbit, itself, as a measure of J_2?

CIUFOLINI: Unfortunately the nodel precession and the rates of change with time of the other orbital parameters are due, not only to the earth quadrupole moment, but to other harmonics too. A solution would be to orbit several high-altitude, laser-ranged satellites, of LAGEOS type, to measure each even zonal harmonic coefficient, to the proper order, plus one satellite to measure the Lense-Thirring effect. Another solution is to orbit polar satellites (Yilmaz 1959, Everitt and Van Patten 1976), since they have a null classical precession. Another solution is to use supplementary inclination satellites to cancel out the classical precession (LAGEOS III, Ciufolini 1984).

SHAPIRO: (Comment on question by K. Nordtvedt). The even zonal harmonics (J_2, J_4, ...) affect other orbital parameters, as well as W, and these effects for the orbit of LAGEOS I, as well as for other satellites, allow estimates to be made of the values of J_2, J_4, ... It was to the residual errors, or uncertainties, from these estimates that Ignazio referred.